Strong-Disorder Renormalization Group Approach to Anderson Localization

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Anderson Localization: waves in random media

Experiments GaMnAs (close to localization)

Extended waves

Localized waves (bound states)

Experiments on (random) optical lattices
Anderson transition

Extended states

Critical states

Localized states

Disorder
Anderson (quantum phase?) transition

\[ \rho_i(\omega) = \sum_n \delta(\omega - \omega_n)|\psi_n(i)|^2 \]

\[ \rho_{av} = \langle \rho_i \rangle \sim 1/W^2 \quad \text{(remains finite)} \]

\[ \rho_{typ} = \exp\{\langle \ln \rho_i \rangle\} \sim (W_c - W)^\beta \]

LOCAL order parameter?
How to tackle Phase Transitions?

**Standard critical points:**

**Spontaneous symmetry breaking**

Order parameter, Landau-Ginzburg

Renormalization group, field theory

**Metal-Insulator Transitions:**

**NO symmetry breaking!**

Order parameter???

"...orthodox phase transition theory will be of little help to us for the time being, and thus the great body of literature in the field is simply irrelevant..."

*Ill–Condensed Matter, 1979*
Can’t decide
(quantum critical region)
Scaling Theory of Localization

Abrahams, Anderson, Licciardello, Ramakrishnana, 1979

Scale-dependent conductance

$$g = F\left(\frac{L}{\xi}\right)$$

$$\beta(g) = \frac{d \ln(g)}{d \ln(L)}$$

All states localized in $d = 1, 2$

Transition at weak disorder in $d = 2 + \varepsilon$

Perturbative RG: 2+epsilon expansion [as for O(N) models]

"Shapiro formula":

$$\beta_{sh}(g) = d - 1 - (1 + g) \ln\left(1 + \frac{1}{g}\right)$$
Problems, open questions

Poor convergence of 2+epsilon expansion?

[Also for O(N) magnets; “missing” topological defects? (Chakravaty, Murthy)]
High mobility 2DEG in Silicon (MOSFET)

Strong-Coupling: Mirror Symmetry/“Duality” Scaling?

S. Kravchenko, 1997
Scaling Theory of Two-Dimensional Metal-Insulator Transitions

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(Received 10 April 1997)

We discuss the recently discovered two-dimensional metal-insulator transition in zero magnetic field in the light of the scaling theory of localization. We demonstrate that the observed symmetry relating conductivity and resistivity follows directly from the quantum critical behavior associated with such a

\[ \beta_T(g) = -\frac{d \ln g}{d \ln T} \approx \frac{1}{\nu z} \ln g \]

\[ g(T) \approx g_o \exp\{\delta n(T_o/T)^{1/\nu z}\} \]

\[ g^*(\delta n, T = 1/g^*(-\delta n, T) \]

“strong coupling” - dominated by insulator?
New Results: Universal Feature of MIT Scaling!

Low-mobility 2DEG (D. Popovic, PRL 2015)

\[
\frac{\langle \sigma \rangle}{T^{1.5}} (e^2/\hbar K_{1.5})
\]

\[
T/T_0 \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1}
\]

\[
T_0 (K) \quad 10^{0} \quad 10^{1} \quad 10^{2} \quad 10^{3}
\]

\[
|\delta_n| \quad 10^{-2} \quad 10^{-1}
\]

- Insulating side: \( z_\nu = 2.13 \pm 0.01 \)
- Metallic side: \( z_\nu = 2.13 \pm 0.03 \)

Mirror symmetry
Quantum Griffiths phases and “Infinite Randomness Fixed Points”

- **D. Fisher** (1992): new scenario for *(insulating)* QCPs with disorder *(Ising)*
- Strong Disorder Renormalization Group (SDRG): **EXACT**

**Griffiths phase** *(Till + Huse)*:

- Rare, dilute magnetically ordered cluster tunnels with rate $\Delta(L) \sim \exp\{-AL^d\}$

**Equations**:
- $P(L) \sim \exp\{-\rho L^d\}$
- $P(\Delta) \sim \Delta^{\alpha-1}$; $\chi \sim T^{\alpha-1}$
- $\alpha \to 0$ at QCP *(IRFP)*

*E. Miranda, V. Dobrosavljevic, Reports on Progress in Physics 68, 2337 (2005)*
Quantum insulating magnets + disorder (D. Fisher et al.)

- Ising (and other discrete symm.) models — **YES**
- Heisenberg (and other continuous symm.) — **NO**

More recently (2003-2008)

Dissipative (itinerant?) magnets + disorder

- Ising model: transition **“rounding”** — **NO** (T. Vojta, 2003; Hoyos, Vojta, 2008)
- Heisenberg (and other continuous symm.) — **YES!!!** (T. Vojta, J. Schmalian, J. Hoyos, 2005-2008)
- Heisenberg + RKKY interactions — **cluster glass** — **NO** (V.D. and E. Miranda, 2005; M. Case and V.D. 2007)
The D-dimensional tight-binding model:

\[ H = \sum_i \epsilon_i c_i^+ c_i + \sum_{i,j} t_{ij} (c_i^+ c_j + h.c.) \]

Identifying strongest energy scale of the problem

\[ \Omega = \max \{|\epsilon_i|, |t_{ij}|\} \]

SDRG (Aoki) transformations exact for studying 2-point conductance

\[ \tilde{t} = -\frac{t_1 t_3 \Omega}{\Omega^2 - \epsilon_2 \epsilon_3} \]

\[ \tilde{\epsilon}_j = \epsilon_j + \frac{\epsilon_3 t_1^2 \delta_{j1} + \epsilon_2 t_3^2 \delta_{j4}}{\Omega^2 - \epsilon_2 \epsilon_3} \]

\[ \tilde{\epsilon}_i = \epsilon_i - \frac{t_2^2 \delta_{i1}}{\Omega} \]

\[ g \equiv g_{\text{typ}} = \frac{\langle T \rangle_{\text{geo}}}{1 - \langle T \rangle_{\text{geo}}} \]
A random graph model with N=100, <k>3.0:

d = inf.

Too many NEW bonds!!
Modified SDRG: Eliminating Irrelevant” Terms


- Tendency to increase the coordination number of the effective lattice
- Solution: Irrelevant bonds in the strong disorder limit (model dependent)

- I. A. Kovacs, and F. Igloi on RTFIM:

- G. Refael et. al. on disordered Bose-Hubbard Hamiltonian: Specifying a thresholding parameter such that

- **Anderson model:** Setting a maximum coordination number \( k_{\text{max}} \) per site

- Extra technical optimizations:
  - Heap data structure (speed up finding the maximum energy term process), Adjancy list (memory storage),...
2-Point Conductance Scaling


Works in ALL dimensions
Critical exponent in all dimensions!


- NO (finite upper critical dimension!
- Excellent convergence of 1/d expansion
- Weak-coupling approach misleading and uninformative
Strong Coupling (high d): Mirror Symmetry Range


MSR increases with d, but \( \sim 10 \) even in \( d = 3 \)
Challenges & Future Work

- SDRG - **analytic** solution in $d = 1$ (so far)
- IRFP in $d >> 1$? Analytic solution?
- Perturbative $1/d$ expansion?
- Adding **correlations** (easy?)
- Can interactions change the **Upper Critical Dimension**? Hyperscaling/Wegner Scaling?
- SDRG theory for **Electronic Griffiths Phases**?
To learn more:

http://badmetals.magnet.fsu.edu
(just Google “Bad Metals”)

Book:

Oxford University Press, June 2012

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