Electronic Griffiths Phases and Quantum Criticality at Disordered Mott Transitions

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Collaborators:
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Jose Hoyos (FSU, Sao Paulo)
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Funding: NSF grants:
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Quantum Griffiths phases and IRFP (1990s)

- **D. Fisher** (1992): new scenario for (insulating) QCPs with disorder (Ising)

\[
P(L) \sim \exp\{-\rho L^d\}
\]

\[
P(\Delta) \sim \Delta^{\alpha-1} ; \quad \chi \sim T^{\alpha-1}
\]

\[
\alpha \rightarrow 0 \text{ at QCP (IRFP)}
\]

**Griffiths phase** (Till + Huse):

Rare, dilute magnetically ordered cluster tunnels with rate \( \Delta(L) \sim \exp\{-AL^d\} \)

E. Miranda, V. Dobrosavljevic, Reports on Progress in Physics 68, 2337 (2005)
Quantum Griffiths phases and IRFP (1990s)

- **D. Fisher (1992):** new scenario for *(insulating)* QCPs with disorder (Ising)

Griffiths phase *(Till + Huse)*:

- Rare, dilute magnetically ordered cluster tunnels with rate \( \Delta(L) \sim \exp\{-AL^d\} \)
- \( P(L) \sim \exp\{-\rho L^d\} \)
- \( P(\Delta) \sim \Delta^{\alpha-1} \); \( \chi \sim T^{\alpha-1} \)
- \( \alpha \to 0 \) at QCP *(IRFP)*


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How generic is IRFP?

Quantum insulating magnets + disorder \textit{(D. Fisher et al.)}

- Ising (and other discrete symm.) models – \textbf{YES}
- Heisenberg (and other continuous symm.) – \textbf{NO}

More recently \textit{(2003-2008)}

\textbf{Dissipative (itinerant?) magnets + disorder}

- Ising model: transition \textit{“rounding”} – \textbf{NO} \textit{(T. Vojta, 2003; Hoyos, Vojta, 2008)}
- Heisenberg (and other continuous symm.) – \textbf{YES}!!! \textit{(T. Vojta, J. Schmalian, J. Hoyos, 2005-2008)}
- Heisenberg + RKKY interactions – \textit{cluster glass} – \textbf{NO} \textit{(V.D. and E. Miranda, 2005; M. Case and V.D. 2007)}
Metal-Insulator Transition?

- Example: Doped semiconductors Si:P,B

\[
\sigma(T = 0) \propto (n - n_c)^\mu
\]

Uncompensated Si:P \((\mu \approx 0.5)\)

Compensated Si:P, B \((\mu \approx 1.0)\)

Metal-Insulator Transition?

Sir Neville Mott

P. W. Anderson
2-Fluid Phenomenology: signatures of **Mott physics**

- Strongly localized states dominate the spin response even in the metallic state. Mott physics dominates the insulator.

\[ \chi(T) \sim \frac{C(T)}{T} \sim T^{\alpha - 1} \] \[ (\alpha \approx 0.7 - 0.8) \]
Large-N theory: analytical solution

[Zhou, Jose Hoyos, Dobrosavljević, Miranda., Europhys. Lett (2009)]

RVB decoupling:

\[ H = \sum_{\langle i,j \rangle} J_{ij} S_i \cdot S_j \Rightarrow \sum_{\langle i,j \rangle, \sigma} \left( \Delta_{ij} f_{i\sigma}^\dagger f_{j\sigma} + \text{h.c.} - |\Delta_{ij}|^2 / J_{ij} \right) + i \sum_i \lambda_i \left( \sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} - 1 \right) \]

VB singlet:

\[ \sum_\sigma \left\langle f_{j\sigma}^\dagger f_{i\sigma} \right\rangle = \Delta_{ij} \]

Single occupancy constraint

(no renormalization at large N)
Disorder near Mott transitions: generic phase diagram

- **Fermi-liquid metal**
  - $\chi(T=0)=\chi_0$
- **Non-Fermi-liquid metal**
  - $\chi(T=0)=\infty$
- **Mott-like insulator (MIT)**

Previous studies (motivated by Si:P):

- **Milovanović, Sachdev, and Bhatt, PRL 63, 82 (1989).** Mean-field theory of the disordered Hubbard model

- **V. Dobrosavljević and G. Kotliar, PRL 78, 3943 (1997).** “statDMFT” on Bethe lattice (finite coordination, D=inf.!!)
The Mott transition

\[ H_H = -t \sum_{\langle i,j \rangle, \sigma} \left( c_{i \sigma}^\dagger c_{j \sigma} + c_{j \sigma}^\dagger c_{i \sigma} \right) + U \sum_i c_{i \uparrow}^\dagger c_{i \uparrow} c_{i \downarrow}^\dagger c_{i \downarrow} \]

**Mott insulator**

**Fermi liquid**

**Anti-ferromagnetic insulator**

\((V_{0.989}Cr_{0.011})_2O_3\)

Death of quasi-particles??

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Almost stuck
Almost stuck

Belgrade, October 5, 2000
Large effective mass $m^* \sim \frac{1}{Z}$

Spectrum of electronic states

Gabi Kotliar
Large effective mass $m^* \sim \frac{1}{Z}$

Quasi-particle weight $Z$ as order parameter

Spectrum of electronic states

1992

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Gabi Kotliar

Paul Cezanne
Dynamical Mean-Field Theory

Low Fermi liquid coherence scale

\[ T^* \sim ZT_F \sim T_F/m^* \]
Dynamical Mean-Field Theory

Low Fermi liquid coherence scale

\[ T^* \sim ZT_F \sim T_F/m^* \]
Dynamical Mean-Field Theory

Low Fermi liquid coherence scale

\[ T^* \sim Z T_F \sim T_F/m^* \]
Effects of disorder?

friend or foe?
statDMFT in $D=2$ (aka “quantum cavity method”)

*Eric Andrade, E. Miranda, V.D., PRL 2009*

\[
H = -t \sum_{(i,j),\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + \sum_{i,\sigma} \varepsilon_i c_{i\sigma}^\dagger c_{i\sigma} + U \sum_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow}
\]

\[
P(\varepsilon_i) = \begin{cases} 
  1/W; & |\varepsilon_i| < W/2 \\
  0; & \text{otherwise}
\end{cases}
\]

**statDMFT:** local (though spatially non-uniform) self-energies

Local renormalizations

Each local site is governed by an impurity action:

\[
S_i = \int_0^\beta d\tau d\tau' \sum_\sigma c_{i\sigma}^\dagger(\tau) [\delta(\tau - \tau') \partial_\tau - \Delta_i(\tau - \tau')] c_{i\sigma}(\tau') + U \int_0^\beta d\tau c_{i\uparrow}^\dagger(\tau) c_{i\downarrow}^\dagger(\tau) c_{i\downarrow}(\tau) c_{i\uparrow}(\tau)
\]

$\Delta_i(\tau)$: hybridization to the neighboring sites
• Clean case ($W=0$): Mott metal-insulator transition at $U=U_c$, where $Z \to 0$ (Brinkman and Rice, 1970).

• Fermi liquid approach in which each fermion acquires a quasi-particle renormalization and each site-energy is renormalized:

$$c_i \rightarrow Z_i c_i \quad \Sigma_i (\omega) = (1 - Z_i^{-1}) \omega - \varepsilon_i + \bar{\varepsilon}_i / Z_i$$
For $U \rightarrow U_c(W)$, all $Z_i \rightarrow 0$ vanish (disordered Mott transition).

If we re-scale all $Z_i$ by $Z_0 \sim U_c(W) - U$, we can look at $P(Z_i/Z_0)$.

For $D = \infty$ (DMFT), $P(Z/Z_0)$ - universal form at $U_c$. 

Results in $D = \infty$ (D. Tanasković et al., PRL 2003; M. C. O. Aguiar et al., PRB 2005)
In $D=2$, the environment of each site ("bath") has strong spatial fluctuations.

**New physics**: rare events due to fluctuations and spatial correlations.

Distribution $P(Z/Z_0)$ acquires a low-$Z$ tail:

$$P(Z) \propto Z^{\alpha-1}$$
• Remembering that the local Kondo temperature and

\[ T_{K_i} \propto Z_i \]

\[ \chi_i(T) \sim \frac{1}{T + T_{K_i}} \Rightarrow \langle \chi(T) \rangle \sim \int dT_k \frac{T_{K}^{\alpha-1}}{T + T_K} \sim T^{\alpha-1} \]

Singular thermodynamic response

The exponent \( \alpha \) is a function of disorder and interaction strength. \( \alpha=1 \) marks the onset of singular thermodynamics.

Quantum Griffiths phase

\[ Z_{\text{typ}} = \exp\{ < \ln Z> \} \]
Most characterized Quantum Griffiths phases are precursors of a critical point where the effective disorder is infinite (D. S. Fisher, PRL 69, 534 (1992); PRB 51, 6411 (1995); ...).

\[ P(Z) \propto Z^{\alpha - 1} \]

\[ \alpha \to 0 \Rightarrow \Delta Z \to \infty \]

\[ \alpha^{-1} - \text{variance of log}(Z) \]

Compatible with infinite randomness fixed point scenario.
Replace the environment of given site outside square by uniform (DMFT-CPA) effective medium.

Reduce square size down to DMFT limit.

Rare events due to rare regions with weaker disorder

The rare event is preserved for a box of size $l > 9$: rather smooth profile with a characteristic size.
“Size” of the rare events: a movie

Killing the Mott droplet
“Size” of the rare events: a movie

Killing the Mott droplet
The effective disorder at the Fermi level is given by the distribution of:

\[ \nu_i = \varepsilon_i + \sum_i (\omega = 0) = \bar{\varepsilon}_i / Z_i \]

Width of the \( \nu_i \) distribution

This quantity is strongly renormalized close to the Mott MIT.

\( \nu_i \) is pinned to Fermi level (Kondo resonance)
However, the effect is lost even slightly away from the Fermi energy:

\[ E = E_F - 0.05D \]

The strong disorder effects reflect the wide fluctuations of \( Z_i \)

Similar to high-Tc materials, as seen by STM Experiment: Seamus Davis (2005) Theory: Garg, Trivedi, Randeria (2008)

\[ U = 0.96 U_c; W = 0.375D \]

Generic to the strongly correlated materials?
However, the effect is lost even slightly away from the Fermi energy:

\[ E = E_F - 0.05D \]

\[ v_i (\omega \neq 0) = \varepsilon_i + \Sigma_i (\omega \neq 0) = v_i + \omega (1 - Z_i^{-1}) \]

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Similar to high-Tc materials, as seen by STM
Experiment: Seamus Davis (2005)
Theory: Garg, Trivedi, Randeria (2008))

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Generic to the strongly correlated materials?
Mottness-induced contrast

\[ \frac{U}{U_c} = 0.87 \]

\[ \frac{U}{U_c} = 0.96 \]
Quantum ripples vs. correlations
- analytical insight -

One impurity – **Friedel oscillations**

Interference: quantum corrections

Ballistic: \( \Delta \sigma \sim T \) (d=2)
Diffusive: \( \Delta \sigma \sim \log T \) (d=2)

(Aleiner, 2001)

Weak impurity - analytic (perturbative) solution, numerics - general

Reduce to standard Hartree-Fock results at small U

**Correlated regime and nonlocal terms??**
"Healing" of density fluctuations

\[ u = \frac{U}{U_c} \quad \frac{m}{m^*} \]

<p>| | |</p>
<table>
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<tr>
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<tr>
<td>0.51</td>
<td>0.95</td>
</tr>
<tr>
<td>0.85</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Analytical result:

\[ \delta n_i = -\frac{2(1-u)}{U_c} \left[ \varepsilon_i + \frac{2}{U_c} (1-u)^2 \left[ \Pi^{(0)} \right]^{-1} \varepsilon_j \right] \]

Lindard function

"healing"
Spatially correlated Gutzwiller factors $Z_i$

**Impurity metallization:** local increase of $Z_i$

\[
u = \frac{U}{U_c} \quad \frac{m}{m^*}
\]

- $0.39 \quad 0.85$
- $0.93 \quad 0.13$
- $0.85 \quad 0.08$

\[\delta Z_i \sim \frac{1 - u}{U_c^2} \left( \frac{\pi (1-d)/2}{2(1+d)/2 \xi (d-3)/2} e^{-r_{ij}/\xi} \right)
\]

\[= 4 (1 - u)^3 \left[ \Pi^{(0)} \right]_{ij}^{-1} \right) \varepsilon_j^2
\]

\[\xi = (2z(1-u))^{-1/2}
\]

- Lindard function
- “healing”
- correlation length
Quantum corrections vs. inelastic scattering

2D ballistic regime

\[
\tau_{\text{tr}}^{-1}(T) = \tau_0^{-1} A^2(u) \left\{ 1 + 2 \frac{T}{T_F} \alpha(u) w(T, \gamma(T)) \right\} + \eta \gamma(T),
\]

\[
w(T, \gamma) = \int \frac{dx}{4} \text{Sech}^2 \left[ \frac{x}{2} \right] \text{Re} \left[ \ln \Gamma \left( \frac{1}{2} + \frac{\gamma(T)}{2\pi T} + i \frac{x}{2\pi} \right) \right]
\]

\[
+ \frac{1}{2} \ln(2\pi) + \frac{\gamma(T)}{2\pi T} \ln \left( \frac{T_F}{2\pi T} \right).
\]

Linear T transport only at \( T < T^* \ll T_F \)

Inelastic (electron-electron) scattering

\[
\gamma(T) = C_\gamma \Lambda(u) T_F (T/T_F)^2
\]

\[
\Lambda(u) \sim (1 - u)^{-2}
\]
Quantum corrections vs. inelastic scattering

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Quantum corrections vs. inelastic scattering

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Inelastic (electron-electron) scattering

\[ \gamma(T) = C \Lambda(u) T_F (T/T_F)^2 \]

\[ \Lambda(u) \sim (1-u)^{-2} \]

Linear T transport only at \( T < T^* \ll T_F \)
Conclusions and...

- An electronic Griffiths phase emerges as a precursor to the disordered Mott MIT (non-Fermi liquid metal)

- Strong-correlation-induced healing of disorder at low energies, but very inhomogeneous away

- Infinite randomness fixed point: new type of critical behavior for disordered MIT???

- Transport behavior? Correlations in diffusive regime??

- RVB singlet/Kondo competition!!