Much progress was made towards understanding the equilibrium properties of vortex systems in superconductors, including the discovery of a topologically ordered Bragg glass phase that is stable in the limit of weak quenched disorder. By contrast, little is known of the dynamics of vortex phases or of their fate when driven out of equilibrium. I will describe experiments that employed time resolved transport and Hall microscopy to probe the evolution of the vortex lattice in NbSe$_2$ in response to a driving force. The results are summarized in a dynamic phase diagram consisting of regions with distinctly different response characteristics. In particular, we identified a regime were the moving vortex lattice exhibits the hallmarks of glassy dynamics: memory, aging and stretched exponential response.