Novel topologically ordered phases of condensed matter

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Thanks

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References

A. M. Essin, J. E. Moore, and D. Vanderbilt, PRL 2009.

“The birth of topological insulators”

Technical reviews by Hasan and Kane (RMP colloquium) and 3D review by Hasan and JEM.
1. Are there topological phases in 3D materials and no applied field?
   Yes — “topological insulators”
   (experimental confirmation 2007 for 2D, 2008 for 3D)
   What makes an insulator topological?

2. Applications
   A. What is the physical response that characterizes a topological insulator?
      “Axion electrodynamics” and magnetoelectric coupling in solids
   B. Magnetotransport puzzles in TIs
   C. Spintronics and thermoelectricity

3. Some open problems: superconductivity and Landau-Ginzburg theory
Types of order

Much of condensed matter is about how different kinds of order emerge from interactions between many simple constituents.

Until 1980, all ordered phases could be understood as “symmetry breaking”:

an ordered state appears at low temperature when the system spontaneously loses one of the symmetries present at high temperature.

Examples:
- **Crystals** break the *translational* and *rotational* symmetries of free space.
- The “liquid crystal” in an LCD breaks *rotational* but not *translational* symmetry.
- **Magnets** break time-reversal symmetry and the rotational symmetry of spin space.
- **Superfluids** break an internal symmetry of quantum mechanics.
Types of order

At high temperature, entropy dominates and leads to a disordered state. At low temperature, energy dominates and leads to an ordered state.

In case this sounds too philosophical, there are testable results that come out of the “Landau theory” of symmetry-breaking:

\[
\rho_L - \rho_G \sim \left(\frac{T_C - T}{T_C}\right)^\beta
\]

Experiment: \( \beta = 0.322 \pm 0.005 \)
Theory: \( \beta = 0.325 \pm 0.002 \)

“Universality” at continuous phase transitions (Wilson, Fisher, Kadanoff, ...)
Types of order

In 1980, the first ordered phase beyond symmetry breaking was discovered.

Electrons confined to a plane and in a strong magnetic field show, at low enough temperature, plateaus in the “Hall conductance”:

- force $I$ along $x$ and measure $V$ along $y$
- on a plateau, get

$$\sigma_{xy} = n \frac{e^2}{h}$$

at least within 1 in $10^9$ or so.

What type of order causes this precise quantization?

Note I: the AC Josephson effect between superconductors similarly allows determination of $e/h$.
Note II: there are also fractional plateaus, about which more later.
Topological Insulators from Spin-orbit Coupling
Semiclassical picture

1D edge of Quantum Hall Effect

1D edge of “Quantum Spin Hall Effect” (Murakami, Nagaosa, Zhang ’04; Kane and Mele ’05; Bernevig et al. ’06)
Experiment: Molenkamp group ‘07

2D surface of 3D Topological Insulator (JEM and Balents, Fu-Kane-Mele, Roy, ’07)
Experiment: Hasan group ’08
Topological order

What type of order causes the precise quantization in the Integer Quantum Hall Effect (IQHE)?

Definition I:

In a topologically ordered phase, some physical response function is given by a “topological invariant”.

What is a topological invariant? How does this explain the observation?

Definition II:

A topological phase is insulating but always has metallic edges/surfaces when put next to vacuum or an ordinary phase.

What does this have to do with Definition I?

“Topological invariant” = quantity that does not change under continuous deformation
Topological invariants

Most topological invariants in physics arise as integrals of some geometric quantity.

Consider a two-dimensional surface.

At any point on the surface, there are two radii of curvature. We define the signed “Gaussian curvature” \( \kappa = (r_1 r_2)^{-1} \)

Now consider closed surfaces.

The area integral of the curvature over the whole surface is “quantized”, and is a topological invariant (Gauss-Bonnet theorem).

\[
\int_M \kappa \, dA = 2\pi \chi = 2\pi \left( 2 - 2g \right)
\]

where the “genus” \( g = 0 \) for sphere, 1 for torus, \( n \) for “n-holed torus”.

from left to right, equators have negative, 0, positive Gaussian curvature
Topological invariants

Good news:
for the invariants in the IQHE and topological insulators,
we need one fact about solids

Bloch’s theorem:
One-electron wavefunctions in a crystal
(i.e., periodic potential) can be written

\[ \psi(r) = e^{i k \cdot r} u_k(r) \]

where \( k \) is “crystal momentum” and \( u \) is periodic (the same in every unit cell).

Crystal momentum \( k \) can be restricted to the Brillouin zone, a region of \( k \)-space
with periodic boundaries.
As \( k \) changes, we map out an “energy band”. Set of all bands = “band structure”.

The Brillouin zone will play the role of the “surface” as in the previous example,
and one property of quantum mechanics, the Berry phase

which will give us the “curvature”.

\[ \psi(r) = e^{i k \cdot r} u_k(r) \]
What kind of “curvature” can exist for electrons in a solid?

Consider a quantum-mechanical system in its (nondegenerate) ground state.

The adiabatic theorem in quantum mechanics implies that, if the Hamiltonian is now changed slowly, the system remains in its time-dependent ground state.

But this is actually very incomplete (Berry).

When the Hamiltonian goes around a closed loop $k(t)$ in parameter space, there can be an irreducible phase

$$\phi = \oint A \cdot dk,$$

$$A = \langle \psi_k | - i \nabla_k | \psi_k \rangle$$

relative to the initial state.

Why do we write the phase in this form? Does it depend on the choice of reference wavefunctions?
Berry phase

Why do we write the phase in this form? Does it depend on the choice of reference wavefunctions?

\[ \phi = \int A \cdot dk, \quad A = \langle \psi_k | - i \nabla_k | \psi_k \rangle \]

If the ground state is non-degenerate, then the only freedom in the choice of reference functions is a local phase:

\[ \psi_k \rightarrow e^{i \chi(k)} \psi_k \]

Under this change, the “Berry connection” \( A \) changes by a gradient,

\[ A \rightarrow A + \nabla_k \chi \]

*just like the vector potential in electrodynamics.*

So loop integrals of \( A \) will be gauge-invariant, as will the *curl* of \( A \), which we call the “Berry curvature”.

\[ \mathcal{F} = \nabla \times A \]
Berry phase in solids

In a solid, the natural parameter space is electron momentum.

The change in the electron wavefunction within the unit cell leads to a Berry connection and Berry curvature:

$$\psi(r) = e^{i \mathbf{k} \cdot \mathbf{r}} u_{\mathbf{k}}(r)$$

$$\mathbf{A} = \langle u_{\mathbf{k}} | - i \nabla_{\mathbf{k}} | u_{\mathbf{k}} \rangle$$

$$\mathbf{F} = \nabla \times \mathbf{A}$$

We keep finding more physical properties that are determined by these quantum geometric quantities.

The first was that the integer quantum Hall effect in a 2D crystal follows from the integral of $F$ (like Gauss-Bonnet!). Explicitly,

$$n = \sum_{\text{bands}} \frac{i}{2\pi} \int d^2 k \left( \left\langle \frac{\partial u}{\partial k_1} \right| \frac{\partial u}{\partial k_2} \right) - \left\langle \frac{\partial u}{\partial k_2} \right| \frac{\partial u}{\partial k_1} \right)$$

$$\mathbf{F} = \nabla \times \mathbf{A}$$

$$\sigma_{xy} = n \frac{e^2}{\hbar}$$

TKNN, 1982    “first Chern number”
The importance of the edge

But wait a moment...

This invariant exists if we have energy bands that are either full or empty, i.e., a “band insulator”.

How does an insulator conduct charge?

Answer: (Laughlin; Halperin)

There are metallic edges at the boundaries of our 2D electronic system, where the conduction occurs.

These metallic edges are “chiral” quantum wires (one-way streets). Each wire gives one conductance quantum ($e^2/h$).

The topological invariant of the bulk 2D material just tells how many wires there have to be at the boundaries of the system.

How does the bulk topological invariant “force” an edge mode?
The importance of the edge

The topological invariant of the bulk 2D material just tells how many wires there have to be at the boundaries of the system.

How does the bulk topological invariant “force” an edge mode?

Answer:

Imagine a “smooth” edge where the system gradually evolves from IQHE to ordinary insulator. The topological invariant must change.

But the definition of our “topological invariant” means that, if the system remains insulating so that every band is either full or empty, the invariant cannot change.

∴ the system must not remain insulating.

(What is “knotted” are the electron wavefunctions)
The same idea will apply in the new topological phases discovered recently:

a “topological invariant”, based on the Berry phase, leads to a nontrivial edge or surface state at any boundary to an ordinary insulator or vacuum.

However, the physical origin, dimensionality, and experiments are all different.

We discussed the IQHE so far in an unusual way. The magnetic field entered only through its effect on the Bloch wavefunctions (no Landau levels!).

This is not very natural for a magnetic field. It is ideal for spin-orbit coupling in a crystal.
The “quantum spin Hall effect”

Spin-orbit coupling appears in nearly every atom and solid. Consider the standard atomic expression

\[ H_{SO} = \lambda \mathbf{L} \cdot \mathbf{S} \]

For a given spin, this term leads to a momentum-dependent force on the electron, somewhat like a magnetic field.

The spin-dependence means that the time-reversal symmetry of SO coupling (even) is different from a real magnetic field (odd).

It is possible to design lattice models where spin-orbit coupling has a remarkable effect: (Murakami, Nagaosa, Zhang 04; Kane, Mele 05)

spin-up and spin-down electrons are in IQHE states, with opposite “effective magnetic fields”. The stability of this phase was explained by Kane and Mele in terms of a “Z2 invariant”.

\[ n=1 \]

Ordinary insulator

2D topological insulator

Ordinary insulator
The “quantum spin Hall effect”

In this type of model, electron spin is conserved, and there can be a “spin current”.

An applied electrical field causes oppositely directed Hall currents of up and down spins.

The charge current is zero, but the “spin current” is nonzero, and even quantized!

However...

1. In real solids there is no conserved direction of spin.

2. So in real solids, it was expected that “up” and “down” would always mix and the edge to disappear.

3. The theory of the above model state is just two copies of the IQHE.
The 2D topological insulator

It was shown in 2005 (Kane and Mele) that, in real solids with all spins mixed and no “spin current”, something of this physics does survive.

In a material with only spin-orbit, the “Chern number” mentioned before always vanishes.

Kane and Mele found a new topological invariant in time-reversal-invariant systems of fermions.

But it isn’t an integer! It is a Chern parity (“odd” or “even”), or a “Z2 invariant”.

Systems in the “odd” class are “2D topological insulators”

1. Where does this “odd-even” effect come from?
2. What is the Berry phase expression of the invariant?
3. How can this edge be seen?
I. Where does this “odd-even” effect come from?

In a time-reversal-invariant system of electrons, all energy eigenstates come in degenerate pairs.

The two states in a pair cannot be mixed by any T-invariant perturbation. (disorder)

So an edge with a single Kramers pair of modes is perturbatively stable (C. Xu-JEM, C. Wu et al., 2006).
The 2D topological insulator

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2. What is the Berry phase expression of the invariant?

It is an integral over half the Brillouin zone,

\[ D = \frac{1}{2\pi} \left[ \oint_{\partial(EBZ)} dk \cdot A - \int_{EBZ} d^2k \mathcal{F} \right] \mod 2 \]  (1)

3. How can this edge be seen?
The 2D topological insulator

Key: the topological invariant predicts the “number of quantum wires”.

While the wires are not one-way, so the Hall conductance is zero, they still contribute to the ordinary (two-terminal) conductance.

There should be a low-temperature edge conductance from one spin channel at each edge:

\[ G = \frac{2e^2}{h} \]

König et al., Science (2007)

This appears in (Hg,Cd)Te quantum wells as a quantum Hall-like plateau in zero magnetic field.
What about 3D?

There is no truly 3D quantum Hall effect. There are only layered versions of 2D. (There are 3 “topological invariants”, from xy, yz, and xz planes.)

Trying to find Kane-Mele-like invariants in 3D leads to a surprise: (JEM and Balents, 2007)

1. There are still 3 layered Z2 invariants, but there is a fourth Z2 invariant as well. Hence there are $2^4 = 16$ different classes of band insulators in 3D.

2. The nontrivial case of the fourth invariant is fully 3D and cannot be realized in any model that doesn’t mix up and down spin. (see also R. Roy, 2009)

In 3D, we can take the BZ to be a cube (with periodic boundary conditions):

- think about xy planes
- 2 inequivalent planes look like 2D problem
- $k_z = \pi/a$
- $k_z = 0$
- $k_z = -\pi/a$

3D “strong topological insulators” go from an 2D ordinary insulator to a 2D topological insulator (or vice versa) in going from $k_z=0$ to $k_z=\pm \pi/a$. This is allowed because intermediate planes have no time-reversal constraint.

3. There should be some type of metallic surface resulting from this fourth invariant, and this is easier to picture...
1. This fourth invariant gives a robust 3D “strong topological insulator” whose metallic surface state in the simplest case is a single “Dirac fermion” (Fu-Kane-Mele, 2007)

2. Some fairly common 3D materials might be topological insulators! (Fu-Kane, 2007)

Claim:
Certain insulators will always have metallic surfaces with strongly spin-dependent structure

How can we look at the metallic surface state of a 3D material to test this prediction?
ARPES of topological insulators

First observation by D. Hsieh et al. (Z. Hasan group), Princeton/LBL, 2008.

This is later data on Bi$_2$Se$_3$ from the same group in 2009:

The states shown are in the “energy gap” of the bulk material--in general no states would be expected, and especially not the Dirac-conical shape.
STM of topological insulators

The surface of a simple topological insulator like $\text{Bi}_2\text{Se}_3$ is “1/4 of graphene”: it has the Dirac cone but no valley or spin degeneracies.

Scanning tunneling microscopy image (Roushan et al., Yazdani group, 2009)

STM can see the absence of scattering within a Kramers pair (cf. analysis of superconductors using quasiparticle interference, D.-H. Lee and S. Davis).
Summary c. 2009

1. There are now more than 3 strong topological insulators that have been seen experimentally \((\text{Bi}_x\text{Sb}_{1-x}, \text{Bi}_2\text{Se}_3, \text{Bi}_2\text{Te}_3, \ldots)\).

2. Their metallic surfaces exist in zero field and have the predicted form.

3. These are fairly common bulk 3D materials (and also \(^3\text{He B}\)).

4. The temperature over which topological behavior is observed can extend up to room temperature or so.

What’s left

What is the physical effect or response that defines a topological insulator beyond single electrons?

What are some “topological” applications?

What kind of order is in topological insulators?
Electrodynamics in insulators

We know that the constants $\varepsilon$ and $\mu$ in Maxwell’s equations can be modified inside an ordinary insulator.

Particle physicists in the 1980s considered what happens if a 3D insulator creates a new term (“axion electrodynamics”, Wilczek 1987)

$$\Delta L_{EM} = \frac{\theta e^2}{2\pi h} \mathbf{E} \cdot \mathbf{B} = \frac{\theta e^2}{16\pi h} \varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}.$$

This term is a total derivative, unlike other magnetoelectric couplings.

The angle $\theta$ is periodic and odd under $T$.

A T-invariant insulator can have two possible values: 0 or $\pi$.

These correspond to “positive” and “negative” Dirac mass for the electron (Jackiw-Rebbi, Callan-Harvey, ...).
Axion E&M, then and now

\[ \Delta L_{EM} = \frac{\theta e^2}{2\pi \hbar} \mathbf{E} \cdot \mathbf{B} = \frac{\theta e^2}{16\pi \hbar} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}. \]

This explains a number of properties of the 3D topological insulator when its surfaces become gapped by breaking T-invariance:

Magnetoelectric effect:
applying B generates polarization P, applying E generates magnetization M)

\[ \sigma_{xy} = (n + \frac{\theta}{2\pi}) \frac{e^2}{\hbar} \quad j \bigcirc \]

Topological insulator slab

\[ \sigma_{xy} = (m - \frac{\theta}{2\pi}) \frac{e^2}{\hbar} \quad j \bigotimes \]
Topological response

Idea of “axion electrodynamics in insulators”

there is a “topological” part of the magnetoelectric term

\[ \Delta \mathcal{L}_{EM} = \frac{\theta e^2}{2\pi \hbar} \mathbf{E} \cdot \mathbf{B} = \frac{\theta e^2}{16\pi \hbar} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}. \]

that is measured by the orbital magnetoelectric polarizability

\[ \theta \frac{e^2}{2\pi \hbar} = \frac{\partial M}{\partial E} = \frac{\partial}{\partial E} \frac{\partial}{\partial B} H = \frac{\partial P}{\partial B} \]

and computed by integrating the “Chern-Simons form” of the Berry phase

\[ \theta = -\frac{1}{4\pi} \int_{BZ} d^3k \, \epsilon_{ijk} \text{Tr}[A_i \partial_j A_k - i\frac{2}{3} A_i A_j A_k] \]  

(Qi, Hughes, Zhang, 2008; Essin, JEM, Vanderbilt 2009)

This integral is quantized only in T-invariant insulators, but contributes in all insulators.
Topological response

Many-body definition: the Chern-Simons or second Chern formula does not directly generalize. However, the quantity $dP/dB$ does generalize: a clue is that the “polarization quantum” combines nicely with the flux quantum.

$$\frac{\Delta P}{B_0} = \frac{e/\Omega}{h/e\Omega} = e^2/h.$$ 

So $dP/dB$ gives a bulk, many-body test for a topological insulator.

(Essin, JEM, Vanderbilt, PRL 2009)
Numerically implemented for small Hubbard models (unpub.)

$e^2/h =$ contact resistance in 0D or 1D
= Hall conductance quantum in 2D
= magnetoelectric polarizability in 3D

For TIs in other AZ classes (Schnyder, Ryu, Furusaki, Ludwig ’08; Kitaev ’08), see Ryu, JEM, Ludwig, arXiv (2010).
Orbital magnetoelectric polarizability

One mysterious fact about the previous result:

We indeed found the “Chern-Simons term” from the semiclassical approach.

But in that approach, it is not at all clear why this should be the only magnetoelectric term from orbital motion of electrons.

More precisely: on general symmetry grounds, it is natural to decompose the tensor into trace and traceless parts

\[
\frac{\partial P_i}{\partial B^j} = \frac{\partial M_j}{\partial E_i} = \alpha^i_j = \tilde{\alpha}^i_j + \alpha_{\theta} \delta^i_j.
\]

The traceless part can be further decomposed into symmetric and antisymmetric parts. (The antisymmetric part is related to the “toroidal moment” in multiferroics; cf. M. Fiebig and N. Spaldin)

But consideration of simple “molecular” models shows that even the trace part is not always equal to the Chern-Simons formula...
Computing orbital \(dP/dB\) in a fully quantum treatment reveals that there are additional terms in general. (Essin et al., 1002.0290)

For \(dM/dE\) approach and numerical tests, see Malashevich, Souza, Coh, Vanderbilt, 1002.0300.

The “ordinary part” indeed looks like a Kubo formula of electric and magnetic dipoles.

Not inconsistent with previous results:
in topological insulators, time-reversal means that only the Berry phase term survives.

There is an “ordinary part” and a “topological part”, which is scalar but is the only nonzero part in TIs. But the two are not physically separable in general.
Both parts are nonzero in multiferroic materials.
Application II: Spintronics

- Observation of giant spin-charge coupling

**Spin-charge coupling**

Charge current = spin density
About 100 times larger than QWs

Align 10 spins/micron\(^2\) $\Rightarrow$ 1 microamp

The locking of spin and momentum at a TI surface means that a charge *current* at one surface generates a spin *density*.

Similarly a charge *density* is associated with a spin *current*.

While these effects could cancel out between the top and bottom surfaces of an unbiased thin film, any asymmetry (such as electrical bias or substrate effects) leads to a net spin-charge coupling.

(O. Yazyev, JEM, S. Louie, PRL 2011)
Goals with current materials

• Currently sought: observation of giant spin-charge coupling

First-principles calculations of surface states, including reduced spin polarization (O. Yazyev, JEM, S. Louie, PRL 2011)

1. Gives numerical strength of spin-charge coupling, e.g., in “inverse spin-galvanic effect” (Garate and Franz): use TI surface current to switch an adsorbed magnetic film

2. Can bias electrically so that combination of surfaces has net spin-charge coupling
Thermoelectric materials can convert waste heat to electrical energy. The 3D topological insulators discovered so far are all useful thermoelectric materials!
Topological insulators and energy

Thermoelectric cooling: refrigeration with no moving parts.

Most consumer thermoelectrics use Bi$_2$Te$_3$, a topological insulator.

Cuisinart CWC-600 6-Bottle Private Reserve Wine Cellar
6 Bottle Wine Cooler:
FRYS.com #: 5049475
Protect the integrity of your favorite wines with the Cuisinart Private Reserve Wine Cellar. This elegant countertop cellar chills wines using thermoelectric cooling technology, which eliminates noise and vibration. Eight temperature presets for a variety of reds and whites keep up to 6 bottles of wine at the perfect serving temperature. Designed in the style of full-size cellars, with a stainless steel door and interior light, the Cuisinart Private Reserve is a beautiful way to display wines and champagnes.
Topological insulators and energy

What makes a material a good thermoelectric?
The “thermoelectric figure of merit” $ZT$ determines Carnot efficiency:

$$ZT = \frac{S^2 \sigma T}{k}$$

$S = V / \Delta T$

“Seebeck coefficient”

$\text{Bi}_2\text{Te}_3$
Topological insulators and energy

So why aren’t thermoelectrics everywhere? Will they be soon?

“(Gordon) Moore’s Law”: (1965-present)
The number of transistors on an IC doubles every 2 years

The thermoelectric figure of merit doubles every 50 years
Topological insulators and energy

Big question: Does knowing that Bi$_2$Te$_3$ has these unusual surface states help with thermoelectric applications?

Yes, at least for low temperature (10K - 77K), where ZT=1 is not currently possible. We hope to double ZT.

Thermoelectrics work best when the band gap is about 5 times $kT$. Gap of Bi$_2$Te$_3$ = 1800 K = 0.15 eV.

Idea: in a thin film, the top and bottom surfaces of a topological insulator “talk” to each other, and a controllable thickness-dependent gap opens.

Key: good thickness and Fermi-level control

Recent development: exfoliated thin films (Balandin et al., UC Riverside, APL)

Fermi-level control in crystals (Hsieh et al., 2009)
Last part: puzzles

1. There is a Berry phase effect in a TI nanowire that is similar but different from the corresponding effect in carbon nanotubes (cf. T. Ando). This has not yet been observed experimentally but efforts continue.

2. Initially, the main materials challenge was to produce genuinely insulating TIs. Considerable progress has been made (Bi$_2$Te$_2$Se/Bi$_2$TeSe$_2$). Surface state mobility > 10,000.

A current challenge: where is the Zeeman effect of the surface states? The effect of the Zeeman term is different than in graphene, since it does not commute with the rest of H.

It is also larger since bulk g >= 20.

3. What kind of order exists in topological insulators? What is the equivalent of Ginzburg-Landau theory of superconductors?
Puzzles in transport

For observation of the above in existing TIs, reduction of bulk residual conductivity is important and seems to be underway.

Magnetic field experiments can isolate 2D surface state features.

**Puzzle 1: Stanford nanowire experiment (Yi Cui et al., Nature Materials)**

sees Aharonov-Bohm \((h/e)\) oscillations, as expected for a clean system, rather than Sharvin & Sharvin \((h/2e)\), as expected for a

The *sign* is also not what is expected in the strong-disorder limit

(Bardarson, Brouwer, JEM, PRL 2010; Zhang and Vishwanath, arXiv 1005.3042).

Puzzle 1 is a *flux* effect. What about *field* measurements?
One puzzle: Where is the Zeeman effect in surface-state magnetoconductance? Key difference between TIs and either graphene or a 2DEG. An alternate mechanism to E-dependent velocity. Or do these cancel? (Why is it so much simpler than in HgTe?)

Landau Levels with Zeeman Coupling

Surface effective Hamiltonian

\[ H = v \left( \sigma^x \pi_y - \sigma^y \pi_x \right) - \frac{g \mu_B}{2} \mathbf{B} \cdot \mathbf{\sigma} \quad (\pi = p + eA) \]

Landau Levels

\[ E_{\pm n} = \begin{cases} \frac{g \mu_B}{2} |B_z| & n = 0 \\ \pm \sqrt{2n \hbar v^2 e |B_z| + \left( \frac{g \mu_B}{2} |B_z| \right)^2} & n > 0 \end{cases} \]
Some challenges:

Materials: Going to Bi$_2$Te$_2$Se is found to alleviate the residual conductivity problems of Bi$_2$Se$_3$ and Bi$_2$Te$_3$. The surface mobility is approaching levels sufficient for FQHE behavior.

Adding copper to Bi$_2$Se$_3$ produces a (topological?) superconductor with $T_c$ around 4 K.

Probes: Still no observation of predicted spin effects in transport.

Theory:
1. What is the equivalent of Landau-Ginzburg theory for TIs?

2. Are there “fractional” topological insulators that are not adiabatically connected to the noninteracting ones?

(2D: Levin and Stern; Partons: Qi et al; Swingle et al.; BF theory; Cho and JEM)
Topological quantum computing

It turns out that the core of a magnetic vortex in a two-dimensional \(p+ip\) superconductor can have a Majorana fermion. (But we haven’t found one yet.)

However, a superconducting layer with this property exists at the boundary between a 3D topological insulator and an ordinary 3D superconductor (Fu and Kane, 2007).
A natural question is whether the surface of a Z2 topological insulator is stable beyond single-particle models.

Time-reversal-breaking perturbations (coupling to a magnetic material or magnetic field) certainly can gap the surface modes.

What about coupling to a superconductor?

Idea: an s-wave proximity effect term

\[ H = \sum_k \left( \Delta c_{k \uparrow} c_{-k \downarrow} + h.c. \right) \]

couples within the low-energy chiral fermion, and hence gives a “spinless” p-wave superconductor (Fu and Kane, PRL 2007).
Conclusions

1. “Topological insulators” exist in two and three dimensions in zero magnetic field.

2. The 3D topological insulator generates a quantized magnetoelectric coupling

\[ \Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi \hbar} \mathbf{E} \cdot \mathbf{B} = \frac{\theta e^2}{16\pi \hbar} \varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}. \]

This is related to a “topological” part of the magnetoelectric coupling in general insulators.

3. These insulators might be useful for magnetoelectrics, quantum computing, spintronics, .... (big leaps) or improved thermoelectrics (not so big a leap).
Some remaining puzzles

Materials:
For a long time, topological insulators weren’t very insulating.
Bi2Te2Se seems to be genuinely insulating in bulk

Adding copper to Bi2Se3 produces a superconductor--is this “topological” like 3He?

Transport: The magnetic field dependence of transport
Puzzles in transport observation of Berry phase

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(Bardarson, Brouwer, JEM, PRL 2010; Zhang and Vishwanath, arXiv 1005.3042).

Puzzle 1 is a flux effect. What about field measurements?
1. There is a Berry phase effect in a TI nanowire that is similar but different from the corresponding effect in carbon nanotubes (cf. T. Ando). This has not yet been observed experimentally but efforts continue.

2. Initially, the main materials challenge was to produce genuinely insulating TIs. Considerable progress has been made (Bi$_2$Te$_2$Se/Bi$_2$TeSe$_2$). Surface state mobility > 10,000.

   A current challenge: where is the Zeeman effect of the surface states? The effect of the Zeeman term is different than in graphene, since it does not commute with the rest of $H$.

   It is also larger since $bulk \ g \geq 20$.

3. What kind of order exists in topological insulators? What is the equivalent of Ginzburg-Landau theory of superconductors?
For observation of the above in existing TIs, reduction of bulk residual conductivity is important and seems to be underway.

Magnetic field experiments can isolate 2D surface state features.

**Puzzle 1: Stanford nanowire experiment (Yi Cui et al., Nature Materials)**

sees Aharonov-Bohm (h/e) oscillations, as expected for a clean system, rather than Sharvin & Sharvin (h/2e), as expected for a

The sign is also not what is expected in the strong-disorder limit

(Bardarson, Brouwer, JEM, PRL 2010; Zhang and Vishwanath, arXiv 1005.3042).

Puzzle 1 is a flux effect. What about field measurements?
Puzzle 2: Where is the Zeeman effect in surface-state magnetoconductance?
Key difference between TIs and either graphene or a 2DEG.
An alternate mechanism to E-dependent velocity

Landau Levels with Zeeman Coupling

Surface effective Hamiltonian
\[ H = v(\sigma^x \pi_y - \sigma^y \pi_x) - \frac{g\mu_B}{2} \mathbf{B} \cdot \mathbf{\sigma} \quad (\pi = p + eA) \]

Landau Levels
\[ E_{\pm n} = \begin{cases} \frac{g\mu_B}{2} |B_z| & n = 0 \\ \pm \sqrt{2n\hbar v^2 e |B_z| + \left(\frac{g\mu_B}{2} |B_z|\right)^2} & n > 0 \end{cases} \]
Field theory of QHE

How can we describe the topological order in the quantum Hall effect, in the way that Landau-Ginzburg theory describes the order in a superconductor?

Standard answer: Chern-Simons Landau-Ginzburg theory

\[ L_{CS} = -\frac{k}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + j^\mu a_\mu, \quad j^\mu = \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda \]

There is an “internal gauge field” \( a \) that couples to electromagnetic \( A \).

Integrating out the internal gauge field \( a \) gives a Chern-Simons term for \( A \), which just describes a quantum Hall effect:

\[ L_{QHE} = -\frac{1}{4k\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \]

There is a difference in principle between the topological field theory and the topological term generated for electromagnetism; they are both Chern-Simons terms.
Topological field theory of QHE

What good is the Chern-Simons theory? (Wen)

\[ L_{CS} = -\frac{k}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + j^\mu a_\mu, \quad j^\mu = \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda \]

The bulk Chern-Simons term is not gauge-invariant on a manifold with boundary.

It predicts that a quantum Hall droplet must have a chiral boson theory at the edge:

\[ S = \frac{k}{4\pi} \int \partial_x \phi (\partial_t \phi - v \partial_x \phi) \, dx \, dt \]

For fractional quantum Hall states, the chiral boson is a “Luttinger liquid” with strongly non-Ohmic tunneling behavior.

Experimentally this is seen qualitatively--perhaps not quantitatively.
Topological field theory of TIs

We believe that in both 2D and 3D the appropriate topological field theory of TIs is a “BF” theory. (G.Y. Cho and JEM, Annals of Physics, to appear)

In 2D this is essentially two copies of Chern-Simons theory. (cf. J. Goryo and collaborators for BF terms in 2D TIs.)

In 3D, it is a bulk topological theory that predicts a single Dirac fermion at the edge.

When the edge is gapped, the magnetoelectric effect results.

\[ L_{BF} = \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda\rho} a_\mu \partial_\nu b_\lambda b_\rho + \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda\rho} A_\mu \partial_\nu b_\lambda b_\rho + C \varepsilon^{\mu\nu\lambda\rho} \partial_\mu a_\nu \partial_\lambda A_\rho \]

Just as in the QHE, one can change coefficients in the topological field theory to obtain a “fractional topological insulator” with a non-Fermi-liquid surface state.

There are fractional statistics of pointlike and linelike excitations.
Statistics in 2D

What makes 2D special for statistics? (Leinaas and Myrheim, 1976)

Imagine looping one particle around another to detect their statistics. In 3D, all loops are equivalent.

In 2D, but not in 3D, the result can depend on the “sense” of the looping (clockwise or counterclockwise). Exchanges are not described by the permutation group, but by the “braid group”.

The effect of the exchange on the ground state need not square to 1. “Anyon” statistics: the effect of an exchange is neither +1 (bosons) or -1 (fermions), but a phase.

Most fractional quantum Hall states, such as the Laughlin state, host “quasiparticles” with anyonic statistics.
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“The birth of topological insulators”
Correlated phases from TI surfaces

Idea of exciton condensation:

(Conventional) Superconductivity occurs when we have identical spin-up and spin-down electron Fermi surfaces and a weak attractive interaction.

Exciton condensation occurs when we have identical electron and hole Fermi surfaces and an attractive interaction between electrons and holes, i.e., Coulomb repulsion.

Why is this difficult? Need an applied field or some other mechanism to keep electrons and holes from recombining.

Alternately can study nonequilibrium condensation before electrons & holes recombine (Butov, Chemla et al.)
Correlated phases from TI surfaces

Formally, exciton condensation is like BCS in the “particle-hole” channel: continuously connected to BEC of excitons.

\[ H_{\text{MF}} = H_0 + (\psi_1^\dagger M \psi_2 + \text{h.c.}) + \frac{1}{U} \text{Tr}(M^\dagger M), \]

Key: unscreened interlayer Coulomb repulsion, with no tunneling.

Generated gap in weak-coupling limit:

\[ m \approx 2\sqrt{V \Lambda} e^{-\Lambda^2/UV} \]

Need large voltage \( V \) and coupling \( U \), with chemical potentials symmetric around Dirac point. New materials (e.g., Ca doping) allow the Dirac point to be moved out of the bulk bandgap.

Transition temperature is of same order or higher than in graphene. Goal: first stable exciton condensate outside quantum Hall regime.