Valence-bond entanglement entropy of random chains of non-Abelian quasiparticles

Huan Tran and N. E. Bonesteel

Department of Physics and NHMFL, Florida State University, 1800 E. P. Dirac, Tallahassee, FL 32310, USA

**Introduction**

- Non-Abelian quasiparticles, described by SU(2)k, Chern-Simons-Witten theory, may exist in certain FQH states (likely candidates are ν = 5/2 and 12/5).
- SU(2)k particles carry quantum numbers called topological charge, which can take values 0, ½, 1, ½k. Topological charge is, in many ways, similar to ordinary spin.
- For a system of $N$ SU(2)k particles, the dimensionality of the Hilbert space of states grows asymptotically as $d^N$ where $d = 2\cos(\pi/k + 2)$ is the quantum dimension of the particles. (For ordinary spin $d = 2$, corresponding to $k \to \infty$).
- If two SU(2)k particles with topological charge ½ fuse, their total topological can be 0 or 1. A singlet is formed if they fuse to 0, a triplet if they fuse to 1.
- A simple Hamiltonian describing a chain of $N$ interacting SU(2)k particles was introduced in [1],

$$H = -\sum J_i \Pi_i^0 \cdot J_i > 0$$

where $J_i$ is singlet-triplet energy splitting of neighboring pairs of particles and $\Pi_i^0$ is the singlet projection operator (See Eq. (2) in next panel).

- The uniform model ($J_i = 1$) was studied in [1] while the random version ($\text{random } J_i$) was studied in [2].
- As for ordinary spin singlets, an SU(2)k singlet can be represented by a valence bond.
- Again, as for ordinary spin, the sector of zero total topological charge of the Hilbert space associated with a chain of SU(2)k particles is spanned by the set of non-crossing valence-bond (VB) states:

- The similarity between this representation of the Hilbert space of a chain of SU(2)k particles and that of ordinary spin-½ particles raises the question: Can the valence-bond Monte Carlo method introduced for spin-½ particles in [3] be used to study the random chains of SU(2)k particles described by Eq. (1)? The answer is “yes”.

**Valence-bond Monte Carlo**

- The ground state $|\text{GS}\rangle$ of $H$ can be projected out from an arbitrary state $|\psi\rangle$ [3]

$$(-H)^N |\psi\rangle = \sum_{P} W(P_n) S(P_n) |\psi\rangle \propto |\text{GS}\rangle$$

with: $P_n = [a_1, a_2, \ldots, a_N]$, $a_i = 1 \ldots N$ refers to $\prod_{p \neq i} \Pi_i^0$, which is a term of $(-H)^n$,

and $|S(P_n)\rangle = \left(\prod_{p \neq i} \Pi_i^0\right) |\text{GS}\rangle$.

$$W(P_n) = \prod_{p \neq i} J_{a_p} w_{a_p} ; w_{a_p} = 1 \text{ or } 1/d \quad \text{[see Eq. (2)]}$$

Action of the projection operator $\Pi_i^0$ on a given VB state

$$\Pi_i^0 |\psi\rangle = \frac{|\psi\rangle}{|\Pi_i^0|^{-1}}$$

**Valence-bond entanglement entropy**

- A quantum system composed of two parts A and B

$$\rho_{AB} = \text{Tr}_B|\text{GS}\rangle\langle\text{GS}|$$

$$\rho_A = -\text{Tr}_B \log \rho_{AB}$$

Reduces density matrix von Neumann (vN) entanglement entropy

- vN entanglement entropy of a spin in a spin-½ singlet is 1, while that for an SU(2)k singlet is $\log d$ [2].
- $S_l$ is the vN entanglement entropy between a block of $L$ particles with the rest of a chain of $N$ particles.

Ref. 2: $\langle\text{GS}\rangle$ of a random chain is frozen into a random singlet phase with

$$S_l = \frac{\ln d}{3} \log_2 L + \text{Constant}; \quad d = 2 \cos\left(\frac{\pi}{k+2}\right)$$

The logarithmic scaling in Eq. (3) becomes that of the random Heisenberg chain as $k \to \infty$ ($d \to 2$), and that of the random transverse Ising model for $k = 2$ ($d = \sqrt{2}$) [4].

- For a given VB state, $S_l = \#$ of singlets crossing the block boundaries $\times$ entanglement entropy per singlet.

For a given VB state, the bond length distribution $n(l)$ is defined as the probability for a bond to have the length $l$.

- From $n(l)$ of the $|\text{GS}\rangle$, which can be computed by VB Monte Carlo, the VB entanglement entropy is defined by [5]

$$S_l = \sum_{l=1}^{L} 2n(l) \times \log_2 (L)$$

- VB entanglement entropy exhibits the logarithmic scaling [Eq. (3)] of vN entanglement entropy of spin-½ systems [5].

**Monte Carlo results**

- For each random configuration (a set of random $J_i$’s), the state obtained by the decimation scheme described in Ref. [2] is chosen to be $|\psi\rangle$.
- $N = 1024, n = 10 N, 5000$ random configurations.

**Conclusions**

- Random chains of non-Abelian quasiparticles can be simulated by the valence-bond Monte Carlo method.
- The ground state of the random chain freezes into a random singlet phase.
- The predicted logarithmic scaling [see Eq. (3)] of the block entanglement entropy of the random chain is confirmed.

**References**

*This work is supported by US DOE*

6. Huan Tran and N. E. Bonesteel, in preparation for publication.